

# NEUTRINO OSCILLATIONS: SOME THEORETICAL IDEAS<sup>1</sup>

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## INTRODUCTION

Over the years, and especially since the discovery of the large mixing of  $\nu_\mu$  seen in atmospheric neutrino experiments, there have been numerous models of neutrino masses proposed in the literature. In the last two years alone, as many as one hundred different models have been published. One of the goals of this talk is to give a helpful classification of these models. Such a classification is possible because in actuality there are only a few basic ideas that underlie the vast majority of published neutrino mixing schemes. After some preliminaries, I give a classification of three-neutrino models, and then in the last part of the talk I discuss in more detail one category of models — those with “lopsided” charged-lepton mass matrices. Finally, I talk about a specific very predictive model based on lopsided mass matrices that I have worked on with Albright and Babu.

## THE DATA

There are four indications of neutrino mass that guide recent attempts to build models: (1) the solar neutrino problem, (2) the atmospheric neutrino anomaly, (3) the LSND experiment, and (4) dark matter. Several excellent reviews of the evidence for neutrino mass have appeared recently.<sup>1</sup>

(1) The three most promising solutions to the solar neutrino problem are based on neutrino mass. These are the small-angle MSW solution (SMA), the large-angle MSW solution (LMA), and the vacuum oscillation solution (VO). All these solutions involve  $\nu_e$  oscillating into some other type of neutrino — in the models we shall consider predominantly  $\nu_\mu$ . In the SMA solution the mixing angle and mass-squared splitting between  $\nu_e$  and the neutrino into which it oscillates are roughly  $\sin^2 2\theta \sim 5.5 \times 10^{-3}$  and  $\delta m^2 \sim 5.1 \times 10^{-6} eV^2$ . For the LMA solution one has  $\sin^2 2\theta \sim 0.79$ , and  $\delta m^2 \sim 3.6 \times 10^{-5} eV^2$ . (The numbers are best-fit values from a recent analysis.<sup>2</sup>) And for the

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VO solution  $\sin^2 2\theta \sim 0.93$ , and  $\delta m^2 \sim 4.4 \times 10^{-10} eV^2$ . (Again, these are best-fit values from a recent analysis.<sup>3)</sup>)

(2) The atmospheric neutrino anomaly strongly implies that  $\nu_\mu$  is oscillating with nearly maximal angle into either  $\nu_\tau$  or a sterile neutrino, with the data preferring the former possibility.<sup>4</sup> One has  $\sin^2 2\theta \sim 1.0$ , and  $\delta m^2 \sim 3 \times 10^{-3} eV^2$ .

(3) The LSND result, which would indicate a mixing between  $\nu_e$  and  $\nu_\mu$  with  $\delta m^2 \sim 0.1 - 1 eV^2$  is regarded with more skepticism for two reasons. The experimental reason is that KARMEN has failed to corroborate the discovery, though KARMEN has not excluded the entire LSND region. The theoretical reason is that to account for the LSND result and also for both the solar and atmospheric anomalies by neutrino oscillations would require three quite different mass-squared splittings, and that can only be achieved with *four* species of neutrino. This significantly complicates the problem of model-building. In particular, it is regarded as not very natural, in general, to have a fourth sterile neutrino that is extremely light compared to the weak scale. For these reasons, the classification given in this talk will assume that the LSND results do not need to be explained by neutrino oscillations, and will include only three-neutrino models.

(4) The fourth possible indication of neutrino mass is the existence of dark matter. If a significant amount of this dark matter is in neutrino mass, it would imply a neutrino mass of order several eVs. In order then to achieve the small  $\delta m^2$ 's needed to explain the solar and atmospheric anomalies one would have to assume that  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  were nearly degenerate. We shall not focus on such models in our classification, which is primarily devoted to models with “hierarchical” neutrino masses. However, in most models with nearly degenerate masses, the neutrino mass matrix consists of a dominant piece proportional to the identity matrix and a much smaller hierarchical piece. Since the piece proportional to the identity matrix would not by itself give oscillations, such models can be classified together with hierarchical mass models in most instances.

In sum, the models we shall classify are those which assume (a) three flavors of neutrino that oscillate ( $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$ ), (b) a hierarchical pattern of neutrino masses, (c) the atmospheric anomaly explained by  $\nu_\mu$ - $\nu_\tau$  oscillations with nearly maximal angle, and (d) the solar anomalies explained by  $\nu_e$  oscillating primarily with  $\nu_\mu$  with either small angle (SMA) or large angle (LMA, VO).

## MAJOR DIVISIONS

There are several major divisions among models. One is between models in which the neutrino masses arise through the see-saw mechanism,<sup>5</sup> and those in which the neutrino masses are generated directly at low energy. In see-saw models, there are both left- and right-handed neutrinos. Consequently, there are five fermion mass matrices to explain: the four Dirac mass matrices,  $U$ ,  $D$ ,  $L$ , and  $N$  of the up quarks, down quarks, charged leptons, and neutrinos, respectively, and the Majorana mass matrix  $M_R$  of the right-handed neutrinos. The four Dirac mass matrices are all roughly of the weak scale, while  $M_R$  is many orders of magnitude larger than the weak scale. After integrating out the superheavy right-handed neutrinos, the mass matrix of the left-handed neutrinos is given by  $M_\nu = -N^T M_R^{-1} N$ . Typically, in see-saw models, the four Dirac mass matrices are closely related to each other, either by grand unification or by flavor symmetries. That means that in see-saw models neutrino masses and mixings are just one aspect of the larger problem of quark and lepton masses, and are likely to shed great light on that problem, and perhaps even be the key to solving it. On the other hand, in most see-saw

models  $M_R$  is either not related or is tenuously related to the Dirac mass matrices of the quarks and leptons. The freedom in  $M_R$  is the major obstacle to making precise predictions of neutrino masses and mixings in most see-saw schemes.

In non-see-saw schemes, there are no right-handed neutrinos. Consequently, there are only four mass matrices to consider, the Dirac mass matrices of the quarks and charged leptons,  $U$ ,  $D$ , and  $L$ , and the Majorana mass matrix of the light left-handed neutrinos  $M_\nu$ . Typically in such schemes  $M_\nu$  has nothing directly to do with the matrices  $U$ ,  $D$ , and  $L$ , but is generated at low-energy by completely different physics.

The three most popular possibilities in recent models for generating  $M_\nu$  at low energy in a non-see-saw way are (a) triplet Higgs, (b) variants of the Zee model,<sup>6</sup> and (c) R-parity violating terms in low-energy supersymmetry. (a) In triplet-Higgs models,  $M_\nu$  arises from a renormalizable term of the form  $\lambda_{ij}\nu_i\nu_jH_T^0$ , where  $H_T$  is a Higgs field in the  $(1, 3, +1)$  representation of  $SU(3) \times SU(2) \times U(1)$ . (b) In the Zee model, the Standard Model is supplemented with a scalar,  $h$ , in the  $(1, 1, +1)$  representation and having weak-scale mass. This field can couple to the lepton doublets  $L_i$  as  $L_iL_jh$  and to the Higgs doublets  $\phi_a$  (if there is more than one) as  $\phi_a\phi_bh$ . Clearly it is not possible to assign a lepton number to  $h$  in such a way as to conserve it in both these terms. The resulting lepton-number violation allows one-loop diagrams that generate a Majorana mass for the left-handed neutrinos. (c) In supersymmetry the presence of such R-parity-violating terms in the superpotential as  $L_iL_jE_k^c$  and  $Q_iD_j^cL_k$ , causes lepton-number violation, and allows one-loop diagrams that give neutrino masses.

It is clear that in all of these schemes the couplings that give rise to neutrino masses have nothing to do with the physics that gives mass to the other quarks and leptons. While this allows more freedom to the neutrino masses, it would from one point of view be very disappointing, as it would mean that the observation of neutrino oscillations is almost irrelevant to the burning question of the origin of quark and charged lepton masses.

Another major division among models has to do with the kinds of symmetries that constrain the forms of mass matrices and that, in some models, relate different mass matrices to each other. There are two main approaches: (a) grand unification, and (b) flavor symmetry. Many models use both.

(a) The simplest grand unified group is  $SU(5)$ . In minimal  $SU(5)$  there is one relation among the Dirac mass matrices, namely  $D = L^T$ , coming from the fact that the left-handed charged leptons are unified with the right-handed down quarks in a  $\mathbf{\bar{5}}$ , while the right-handed charged leptons and left-handed down quarks are unified in a  $\mathbf{10}$ . In  $SU(5)$  there do not have to be right-handed neutrinos, though they may be introduced. In  $SO(10)$ , which in several ways is a very attractive group for unification, the minimal model gives the relations  $N = U \propto D = L$ . In realistic models these relations are modified in various ways, for example by the appearance of Clebsch coefficients in certain entries of some of the mass matrices. It is clear that unified symmetries are so powerful that very predictive models are possible. Most of the published models which give sharp predictions for masses and mixings are unified models.

(b) Flavor symmetries can be either abelian or non-abelian. Non-abelian symmetries are useful for obtaining the equality of certain elements of the mass matrix, as in models where the neutrino masses are nearly degenerate, and in the so-called “flavor democracy” schemes. Abelian symmetries are useful for explaining hierarchical mass matrices through the so-called Froggatt-Nielson mechanism.<sup>7</sup> The idea is that different fermion multiplets can differ in charge under a  $U(1)$  flavor symmetry that is spontaneously broken by some “flavon” expectation value (or values),  $\langle f_i \rangle$ . Thus, different elements of the fermion mass matrices would be suppressed by different powers of  $\langle f_i \rangle/M \equiv \epsilon_i \ll 1$ ,

where  $M$  is the scale of flavor physics. This kind of scheme can explain small mass ratios and mixings in the sense of predicting them to arise at certain orders in the small quantities  $\epsilon_i$ . A drawback of such models compared to many grand unified models, is that actual numerical predictions, as opposed to order of magnitude estimates, are not possible. On the other hand, models based on flavor symmetry involve less of a theoretical superstructure built on top of the Standard Model than do unified models, and could therefore be considered more economical in a certain sense. Unified models put more in but get more out than flavor symmetry.

## THE PUZZLE OF LARGE $\nu_\mu - \nu_\tau$ MIXING

The most significant new fact about neutrino mixing is the largeness of the mixing between  $\nu_\mu$  and  $\nu_\tau$ . This comes as somewhat of a surprise from the point of view of both grand unification and flavor symmetry approaches. Since grand unification relates leptons to quarks, one might expect lepton mixing angles to be small like those of the quarks. In particular, the mixing between the second and third family of quarks is given by  $V_{cb}$ , which is known to be 0.04. That is to be compared to the nearly maximal mixing of the second and third families of leptons:  $U_{\mu 3} \cong 1/\sqrt{2} \cong 0.7$ . It is true that even in the early 1980's some grand unified models predicted large neutrino mixing angles. (Especially noteworthy is the remarkably prophetic 1982 paper of Harvey, Ramond, and Reiss,<sup>8</sup> which explicitly predicted and emphasized that there should be large  $\nu_\mu - \nu_\tau$  mixing. However, in those days the top mass was expected to be light, and Ref. 8 chose it to be 25 GeV. That gave  $V_{cb}$  in that model to be about 0.22. The corresponding lepton mixing was further boosted by a Clebsch of 3. With the actual value of  $m_t$  that we now know, the model of Ref. 8 would predict  $U_{\mu 3}$  to be 0.12). What makes the largeness of  $U_{\mu 3}$  a puzzle in the present situation is the fact that we now know that both  $V_{cb}$  and  $m_c/m_t$  are exceedingly small.

The same puzzle exists in the context of flavor symmetry. The fact that the quark mixing angles are small suggests that there is a family symmetry that is only weakly broken, while the large mixings of some of the neutrinos suggests that family symmetries are badly broken.

The chief point of interest in looking at any model of neutrino mixing is how it explains the large mixing of  $\nu_\mu$  and  $\nu_\tau$ . This will be the feature that I will use to organize the classification of models.

## CLASSIFICATION OF THREE-NEUTRINO MODELS

Virtually all published models fit somewhere in the simple classification now to be described. The main divisions of this classification are based on how the large  $\nu_\mu - \nu_\tau$  mixing arises. This mixing is described by the element  $U_{\mu 3}$  of the so-called MNS matrix (analogous to the CKM matrix for the quarks).

The mixing angles of the neutrinos are the mismatch between the eigenstates of the neutrinos and those of the charged leptons, or in other words between the mass matrices  $L$  and  $M_\nu$ . Thus, there are two obvious ways of obtaining large  $U_{\mu 3}$ : either  $M_\nu$  has large off-diagonal elements while  $L$  is nearly diagonal, or  $L$  has large off-diagonal elements and  $M_\nu$  is nearly diagonal. Of course this distinction only makes sense in some preferred basis. But in almost every model there is some preferred basis given by the underlying symmetries of that model. This distinction gives the first major division in

the classification, between models of what I shall call class I and class II. (It is also possible that the large mixing is due almost equally to large off-diagonal elements in  $L$  and  $M_\nu$ , but this possibility seems to be realized in very few published models. I will put them into class II.)

If the large  $U_{\mu 3}$  is due to  $M_\nu$  (class I), then it becomes important whether  $M_\nu$  arises from a non-see-saw mechanism or the see-saw mechanism. We therefore distinguish these cases as class I(1) and class I(2) respectively. In the see-saw models,  $M_\nu$  is given by  $-N^T M_R^{-1} N$ , so a further subdivision is possible: between models in which the large off-diagonal elements are in  $M_R$  and those in which they are in  $N$ . We call these class I(2A) and I(2B) respectively.

If  $U_{\mu 3}$  is due to large off-diagonal elements in  $L$ , while  $M_\nu$  is nearly diagonal (class II), then the question to ask is why, given that  $L$  has large off-diagonal elements, there are not also large off-diagonal elements in the Dirac mass matrices of the other charged fermions, namely  $U$  and  $D$ , causing large CKM mixing of the quarks. In the literature there seem to be two ways of answering this question. One way involves the CKM angles being small due to a cancellation between large angles that are nearly equal in the up and down quark sectors. We call this class II(1). The main examples of this idea are the so-called “flavor democracy models”. The other idea is that the matrices  $L$  and  $D^T$  (related by unified or flavor symmetry) are “lopsided” in such a way that the large off-diagonal elements only affect the mixing of fermions of one handedness: left-handed for the leptons, making  $U_{\mu 3}$  large, and right-handed for the quarks, leaving  $V_{cb}$  small. We call this approach class II(2).

Schematically, one then has

$$\begin{aligned}
I \quad & \text{Large mixing from } M_\nu \\
& (1) \text{ Non see saw} \\
& (2) \text{ See saw} \\
& \quad \text{A. Large mixing from } M_R \\
& \quad \text{B. Large mixing from } N \\
II \quad & \text{Large mixing from } L \\
& (1) \text{ CKM small by cancellation} \\
& (2) \text{ lopsided } L.
\end{aligned} \tag{1}$$

Now let us examine the different categories in more detail, giving examples from the literature.

### **I(1) Large mixing from $M_\nu$ , non-see-saw.**

This kind of model gives a natural explanation of the discrepancy between the largeness of  $U_{\mu 3}$  and the smallness of  $V_{cb}$ .  $V_{cb}$  comes from Dirac mass matrices, which are all presumably nearly diagonal like  $L$ , whereas  $U_{\mu 3}$  comes from the matrix  $U_\nu$ ; and since in non-see-saw models  $M_\nu$  comes from models the matrix  $M_\nu$  comes from completely different physics than do the Dirac mass matrices it is not at all surprising if it has a very different form from the others, containing some large off-diagonal elements. While this basic idea is very simple and appealing, these models have the drawback that in non-see-saw models the form of  $M_\nu$ , since it comes from new physics unrelated to the origin of the other mass matrices, is highly unconstrained. Thus, there are few definite predictions, in general, for masses and mixings in such schemes. However, in some schemes constraints can be put on the new physics responsible for  $M_\nu$ .

As we saw, there are a variety of attractive ideas for generating a non-see-saw  $M_\nu$  at low energy, and there are published models of neutrino mixing corresponding to all these ideas.<sup>9–13</sup>  $M_\nu$  comes from triplet Higgs in Ref. 9; from the Zee mechanism in Ref. 10; and from R-parity and lepton-number-violating terms in a SUSY model in Ref. 11.

In Ref. 12 a “democratic form” of  $M_\nu$  is enforced by a family symmetry. Several other models in class I(1) exist in the literature.<sup>13</sup>

### I(2A) See-saw $M_\nu$ , large mixing from $M_R$

In these models,  $M_\nu$  comes from the see-saw mechanism and therefore has the form  $-N^T M_R^{-1} N$ . The large off-diagonal elements in  $M_\nu$  are assumed to come from  $M_R$ , while the Dirac neutrino matrix  $N$  is assumed to be nearly diagonal and hierarchical like the other Dirac matrices  $L$ ,  $U$ , and  $D$ . As with the models of class I(1), these models have the virtue of explaining in a natural way the difference between the lepton angle  $U_{\mu 3}$  and the quark angle  $V_{cb}$ . The quark mixings all come from Dirac matrices, while the lepton mixings involve the Majorana matrix  $M_R$ , which it is quite reasonable to suppose might have a very different character, with large off-diagonal elements.

However, there is a general problem with models of this type, which not all the examples in the literature convincingly overcome. The problem is that if  $N$  has a hierarchical and nearly diagonal form, it tends to communicate this property to  $M_\nu$ . For example, suppose we take  $N = \text{diag}(\epsilon', \epsilon, 1)M$ , with  $1 \gg \epsilon \gg \epsilon'$ . And suppose that the  $ij^{\text{th}}$  element of  $M_R^{-1}$  is called  $a_{ij}$ . Then the matrix  $M_\nu$  will have the form

$$M_\nu \propto \begin{pmatrix} \epsilon'^2 a_{11} & \epsilon' \epsilon a_{12} & \epsilon' a_{13} \\ \epsilon' \epsilon a_{12} & \epsilon^2 a_{22} & \epsilon a_{23} \\ \epsilon' a_{13} & \epsilon a_{23} & a_{33} \end{pmatrix}. \quad (2)$$

If all the non-vanishing elements  $a_{ij}$  were of the same order of magnitude, then obviously  $M_\nu$  is approximately diagonal and hierarchical. The contribution to the leptonic angles coming from  $M_\nu$  would therefore typically be proportional to the small parameters  $\epsilon$  and  $\epsilon'$ . This suggests that to get a value of  $U_{\mu 3}$  that is of order 1, it is necessary to have the small parameter coming from  $N$  get cancelled by a correspondingly large parameter from  $M_R^{-1}$ . The trouble is that to have such a conspiracy between the magnitudes of parameters in  $N$  and  $M_R$  is unnatural, in general, since these matrices have very different origins. This problem has been pointed out by various authors.<sup>14</sup> We shall call it the Dirac-Majorana conspiracy problem.

There are several models in the literature that fall into class I(2A).<sup>15–17</sup> Of these, an especially interesting paper is that of Jezabek and Sumino,<sup>15</sup> because it shows that a Dirac-Majorana conspiracy can be avoided. Jezabek and Sumino consider the case that the Dirac and Majorana matrices of the neutrinos have the forms

$$N = \begin{pmatrix} x^2 y & 0 & 0 \\ 0 & x & x \\ 0 & O(x^2) & 1 \end{pmatrix} m_D, \quad M_R = \begin{pmatrix} 0 & 0 & A \\ 0 & 1 & 0 \\ A & 0 & 0 \end{pmatrix} m_R, \quad (3)$$

where  $x$  is a small parameter. If one computes  $M_\nu = -N^T M_R^{-1} N$  one finds that

$$M_\nu = - \begin{pmatrix} 0 & O(x^4 y/A) & x^2 y/A \\ O(x^4 y/A) & x^2 & x^2 \\ x^2 y/A & x^2 & x^2 \end{pmatrix} m_D^2 / m_R. \quad (4)$$

Note that this gives a maximal mixing of the second and third families, without having to assume any special relationship between the small parameter in  $N$  (namely  $x$ ) and the parameter in  $M_R$  (namely  $A$ ). Altarelli and Feruglio<sup>16</sup> generalize this example, showing that the same effect occurs if  $M_R$  is taken to have a triangular symmetric form.

An interesting point about the form of  $M_\nu$  in Eq. (4) is that it gives bimaximal mixing. This is easily seen by doing a rotation of  $\pi/4$  in the 2-3 plane, bringing the matrix to the form

$$M'_\nu = \begin{pmatrix} 0 & z & z' \\ z & 0 & 0 \\ z' & 0 & 2x^2 \end{pmatrix}. \quad (5)$$

In the 1-2 block this matrix has a Dirac form, giving nearly maximal mixing of  $\nu_e$ .

Other published models that fall into class I(2) are given in Ref. 17.

### I(2B) See-saw $M_\nu$ , large mixing from $N$

At least at first glance, this seems to be a less natural approach. The point is that if the large  $U_{\mu 3}$  is due to large off-diagonal elements in  $N$ , it might be expected that the other Dirac mass matrices,  $U$ ,  $D$ , and  $L$ , would also have large off-diagonal elements, giving large CKM angles. There are ways around this objection, and a few interesting models that fall into this class have been constructed. However, experience seems to show that this approach is harder to make work than the others, and fewer models of this type exist in the literature.<sup>18</sup>

### II(1) Large mixing from $L$ , CKM small by cancellation

If the large value of  $U_{\mu 3}$  comes from large off-diagonal elements in the mass matrix  $L$  of the *charged* leptons, then it is most natural to assume that the other Dirac mass matrices have large off-diagonal elements also. Why, then, are the CKM angles small? One possibility is that the CKM angles are small because of an almost exact cancellation between large angles needed to diagonalize  $U$  and  $D$ . That, in turn, would imply that  $U$  and  $D$ , even though highly non-diagonal, have nearly identical forms. This is the idea realized in so-called “flavor democracy” models.

In flavor democracy models, a permutation symmetry  $S_3 \times S_3$  among the left- and right-handed fermions causes the Dirac mass matrices  $L$ ,  $D$ , and  $U$  to have the form

$$L, D, U \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (6)$$

Smaller contributions that break the permutation symmetry cause deviations from this form. These flavor-democratic forms are of rank 1, explaining why one family is much heavier than the others. On the other hand, the mass matrix of the neutrinos  $M_\nu$  is assumed to have, by an  $S_3$  symmetry acting on the left-handed neutrinos, the approximate form

$$M_\nu \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

If  $M_\nu$  were *exactly* proportional to the identity, then the basis of neutrino mass eigenstates would be undefined, and so then would be the MNS angles. However, once the small  $S_3$ -violating effects are taken into account, a neutrino basis is picked out. It is not surprising that, typically, the neutrino angles that are predicted are of order unity. On the other hand, the fact that  $U$  and  $D$  are nearly the same in form leads to a cancellation that tends to make the quark mixing angles small.

Exactly what angles are predicted for the neutrinos depends on the form of the small contributions to the mass matrices that break the permutation symmetries. There are many simple forms that might be assumed, and the possibilities are rich. There exists a large and growing literature on these models.<sup>19</sup>

The idea of flavor democracy is an elegant one, especially in that it uses one basic idea to explain the largeness of the leptonic angles, the smallness of the quark angles,

and the fact that one family is much heavier than the others. On the other hand, it requires the very specific forms given in Eqs. (6) and (7), which come from very specific symmetries. It is in this sense a narrower approach to the problem of fermion masses than some of the others I have mentioned.

It would be interesting to know whether models of class II(1), in which the CKM angles are small by cancellations of large angles, can be constructed using ideas other than flavor democracy.

## II(2) Large mixing from “lopsided” $L$

We now come to what I regard as the most elegant way to explain the largeness of  $U_{\mu 3}$ : “lopsided”  $L$ . The basic idea is that the charged-lepton and down-quark mass matrices have the approximate forms

$$L \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \sigma & 1 \end{pmatrix} m_D, \quad D \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma \\ 0 & \epsilon & 1 \end{pmatrix} m_D. \quad (8)$$

The “ $\sim$ ” sign is used because in realistic models these  $\sigma$  and  $\epsilon$  entries could have additional factors of order unity, such as from Clebsches. The fact that  $L$  is related closely in form to the *transpose* of  $D$  is a very natural feature from the point of view of  $SU(5)$  or related symmetries, and is a crucial ingredient in this approach. The assumption is that  $\epsilon \ll 1$ , while  $\sigma \sim 1$ . In the case of the charged leptons  $\epsilon$  controls the mixing of the second and third families of *right*-handed fermions (which is not observable at low energies), while  $\sigma$  controls the mixing of the second and third families of *left*-handed fermions, which contributes to  $U_{\mu 3}$  and makes it large. For the quarks the reverse is the case because of the “ $SU(5)$ ” feature: the small  $O(\epsilon)$  mixing is in the left-handed sector, accounting for the smallness of  $V_{cb}$ , while the large  $O(\sigma)$  mixing is in the right-handed sector, where it cannot be observed and does no harm.

In this approach the three crucial elements are these: (a) Large mixing of neutrinos (in particular of  $\nu_\mu$  and  $\nu_\tau$ ) caused by large off-diagonal elements in the *charged*-lepton mass matrix  $L$ ; (b) this off-diagonal element appearing in a highly asymmetric or lopsided way; and (c)  $L$  being similar to the transpose of  $D$  by  $SU(5)$  or a related symmetry.

To my knowledge the first place that all the elements of this approach appear is in a paper by Babu and Barr<sup>20</sup> and a sequel by Barr.<sup>21</sup> In those papers the emphasis was on a particular mechanism (in  $SU(5)$  and  $SO(10)$ ) by which the lopsidedness of  $L$  and  $D$  can arise. So perhaps it was not noticed by some readers that the scheme described in those papers was an instance of a more general mechanism.

The next time that this general idea can be found is in three papers that appeared almost simultaneously: Sato and Yanagida,<sup>22</sup> Albright, Babu, and Barr,<sup>23</sup> and Irges, Lavignac, and Ramond.<sup>24</sup>

It is interesting that the same mechanism was arrived at independently by these three groups from completely different points of view. In Sato and Yanagida the model is based on  $E_7$ , and the structure of the matrices is determined by the Froggatt-Nielson mechanism. In Albright, Babu, and Barr, the model was based on  $SO(10)$ , and does not use the Froggatt-Nielson approach. Rather, the constraints on the form of the mass matrices come from assuming a “minimal” set of Higgs for  $SO(10)$  and choosing the smallest and simplest set of Yukawa operators that can give realistic matrices. Though both papers assume a unified symmetry larger than  $SU(5)$ , in both it is the  $SU(5)$  subgroup that plays the critical role in relating  $L$  to  $D^T$ . The model of Irges, Lavignac, and Ramond, like that of Sato and Yanagida, uses the Froggatt-Nielson idea, but is not based on a grand unified group. Rather, the fact that  $L$  is related to  $D^T$



follows ultimately from the requirement of anomaly cancellation for the various  $U(1)$  flavor symmetries of the model. However, it is well known that anomaly cancellation typically enforces charge assignments that can be embedded in unified groups. So that even though the model does not contain an explicit  $SU(5)$ , it could be said to be “ $SU(5)$ -like”.

In the last two years, the same mechanism has been employed by a large number of authors using a variety of approaches.<sup>25</sup>

## A PREDICTIVE $SO(10)$ MODEL WITH LOPSIDED $L$

The model that I shall now describe briefly was not constructed to explain neutrino phenomenology; rather it emerged from the attempt to find a realistic model of the masses of the charged leptons and quarks in the context of  $SO(10)$ . In particular, the idea was to take the Higgs sector of  $SO(10)$  to be as minimal as possible, and then to find what this implied for the mass matrices of the quarks and leptons. In fact, in the first paper we wrote, we did not pay any attention to the neutrino spectrum. Then we noticed that the model in that paper actually predicted a large mixing of  $\nu_\mu$  with  $\nu_\tau$  and published a follow-up paper.<sup>23</sup> The reason for the large mixing of the mu and tau neutrinos was precisely the fact that the charged lepton mass matrix has a lopsided form.

The reason this lopsided form was built into this model (which I shall refer to as the ABB model henceforth) was that it was necessary to account for certain well-known features of the mass spectrum of the quarks. In particular, the mass matrix entry that is denoted  $\sigma$  in Eq. (8) above plays three crucial roles in the ABB model that have nothing to do with neutrino mixing. (1) It is required to get the Georgi-Jarlskog<sup>26</sup> factor of 3 between  $m_\mu$  and  $m_s$ . (2) It explains the value of  $V_{cb}$ . (3) It explains why  $m_c/m_t \ll m_s/m_b$ . Remarkably, it turns out not only to perform these three tasks, but also gives mixing of order 1 between  $\nu_\mu$  and  $\nu_\tau$ . Not often are four birds killed with one stone!

In constructing the model, several considerations guided us. First, we assumed the “minimal” set of Higgs for  $SO(10)$ . It has been shown<sup>27</sup> that the smallest set of Higgs that will allow a realistic breaking of  $SO(10)$  down to  $SU(3) \times SU(2) \times U(1)$ , with natural doublet-triplet splitting,<sup>28</sup> consists of a single adjoint (**45**), two pairs of spinors (**16** +  **$\overline{16}$** ), a pair of vectors (**10**), and some singlets. The adjoint, in order to give the doublet-triplet splitting, must have a VEV proportional to the  $SO(10)$  generator  $B - L$ . This fact is an important constraint. Second, we assumed that the qualitative features of the quark and lepton spectrum should not arise by artificial cancellations or numerical accidents. Third, we required that the Georgi-Jarlskog factor arise in a simple and natural way. Fourth, we assumed that the entries in the mass matrices should come from operators of low-dimension that arise in simple ways from integrating out small representations of fermions.

Having imposed these conditions of economy and naturalness on the model we were led to a structure coming from just six effective Yukawa terms (just five if  $m_u$  is allowed to vanish). These gave the following mass matrices:

$$\begin{aligned}
U^0 &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \frac{1}{3}\epsilon \\ 0 & -\frac{1}{3}\epsilon & 1 \end{pmatrix} m_U, & D^0 &= \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma + \frac{1}{3}\epsilon \\ \delta' & -\frac{1}{3}\epsilon & 1 \end{pmatrix} m_D \\
N^0 &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} m_U, & L^0 &= \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & -\epsilon \\ \delta' & \sigma + \epsilon & 1 \end{pmatrix} m_D.
\end{aligned} \tag{9}$$

(The first papers<sup>23</sup> gave only the structures of the second and third families, while this was extended to the first family in a subsequent paper.<sup>29</sup>) Here  $\sigma \cong 1.8$ ,  $\epsilon \cong 0.14$ ,  $\delta \cong |\delta'| \cong 0.008$ ,  $\eta \cong 0.6 \times 10^{-5}$ . The patterns that are evident in these matrices are due to the  $SO(10)$  group-theoretical characteristics of the various Yukawa terms. Notice several facts about the crucial parameter  $\sigma$  that is responsible for the lopsidedness of  $L$  and  $D$ . First, if  $\sigma$  were not present, then instead of the Georgi-Jarlskog factor of 3, the ratio  $m_\mu/m_s$  would be given by 9. (That is, the Clebsch of  $\frac{1}{3}$  that appears in  $D$  due to the generator  $B - L$  gets squared in computing  $m_s$ .) Since the large entry  $\sigma$  overpowers the small entries of order  $\epsilon$ , the correct Georgi-Jarlskog factor emerges. Second, if  $\sigma$  were not present,  $U$  and  $D$  would be proportional, as far as the two heavier families are concerned, and  $V_{cb}$  would vanish. Third, by having  $\sigma \sim 1$  one ends up with  $V_{cb}$  and  $m_s/m_b$  being of the same order ( $\epsilon$ ) as is indeed observed. And since  $\sigma$  does not appear in  $U$  (for group-theoretical reasons) the ratio  $m_c/m_t$  comes out much smaller, of order  $\epsilon^2$ , also as observed. In fact, with this structure, the mass of charm is predicted correctly to within the level of the uncertainties.

Thus, for several reasons that have nothing to do with neutrinos one is led naturally to the very lopsided form that we found gives an elegant explanation of the mixing seen in atmospheric neutrino data!

From the very small number of Yukawa terms, and from the fact that  $SO(10)$  symmetry gives the normalizations of these terms, and not merely order of magnitude estimates for them, it is not surprising that many precise predictions result. In fact there are altogether nine predictions.<sup>29</sup> Some of these are post-dictions (including the highly non-trivial one for  $m_c$ ). But several predictions will allow the model to be tested in the future, including predictions for  $V_{ub}$ , and the mixing angles  $U_{e2}$   $U_{e3}$ .

In the first papers it appeared that the model only gave the small-angle MSW solution to the solar neutrino problem. In fact, if  $\eta = 0$ , or if forms for  $M_R$  are chosen that do not involve much mixing of the first-family right-handed neutrino with the others, then a very precise prediction for  $U_{e2}$  results that is beautifully consistent with the small-angle MSW solution.<sup>29</sup> However, in a subsequent paper<sup>30</sup> we showed that for other simple forms of  $M_R$  the model gives bi-maximal mixing. (This happens in a way similar to what we saw above in Eqs. (4) and (5) for the Jezabek-Sumino model.)

For more details of the ABB model and its predictions I refer you the papers I have mentioned.

(The classification given in this talk has been somewhat expanded in a paper by Barr and Dorsner.<sup>31</sup> That paper also contains a much more complete listing of three-neutrino models that have been published in the last few years. It also gives a general discussion of expectations for the parameter  $U_{e3}$ .)

## REFERENCES

1. J.W.F. Valle, Neutrino physics at the turn of the millenium (hep-ph/9911224); S.M. Bilenky, Neutrino masses, mixings, and oscillations, Lectures at the 1999

European School of High Energy Physics, Casta Papiernicka, Slovakia, Aug. 22-Sept. 4, 1999 (hep-ph/0001311).

2. M.C. Gozalez-Garcia, P.C. de Holanda, C. Peña-Garay, and J.C.W. Valle, Status of the MSW solutions to the solar neutrino problem, hep-ph/9906469
3. V. Barger and K. Whisnant, Seasonal and energy dependence of solar neutrino vacuum oscillations, hep-ph/9903262
4. M.C. Gonzalez-Garcia, talk at International Workshop on Particles in Astrophysics and Cosmology: From Theory to Observation, Valencia, Spain, May 3-8, 1999.
5. M. Gell-Mann, P. Ramond, and Slansky, in *Supergravity, Proc. Supergravity Workshop at Stony Brook*, ed. P. Van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, *Proc. Workshop on Unified theory and the baryon number of the universe*, ed. O. Sawada and A. Sugamota (KEK, 1979).
6. A. Zee, Phys. Lett. **B93** (1980) 389; Phys. Lett. **B161** (1985) 141.
7. C. Froggatt and H.B. Nielson, Nucl. Phys. **B147** (1979) 277.
8. J.A. Harvey, D.B. Reiss, and P. Ramond, Mass relations and neutrino oscillations in an  $SO(10)$  model, Nucl. Phys. **B199** (1982) 223-268.
9. R.N. Mohapatra and S. Nussinov, Bimaximal neutrino mixing and neutrino mass matrix, Phys. Rev. **D60** (1999) 013002 (hep-ph/9809415).
10. C. Jarlskog, M. Matsuda, S. Skadhauge, and M. Tanimoto, Zee mass matrix and bimaximal neutrino mixing, Phys. Lett. **B449** (1999) 240-252 (hep-ph/9812282).
11. M. Drees, S. Pakvasa, X. Tata, T. terVeldhuis, A supersymmetric resolution of solar and atmospheric neutrino puzzles, Phys. Rev. **D57** (1998) 5335-5339 (hep-ph/9712392).
12. K. Fukuura, T. Miura, E. Takasugi, and M. Yoshimura, Maximal CP violation, large mixings of neutrinos and democratic type neutrino mass matrix, Osaka Univ. preprint, OU-HET-326 (hep-ph/9909415).
13. G.K. Leontaris and J. Rizos, New fermion mass textures from anomalous  $U(1)$  symmetries with baryon and lepton number conservation, CERN-TH-99-268 (hep-ph/9909206).
14. M. Jezabek and Y. Sumino, Neutrino mixing and seesaw mechanism, Phys. Lett. **B440** (1998) 327-331 (hep-ph/9807310); G. Altarelli and F. Feruglio, Neutrino mass textures from oscillations with maximal mixing, Phys. Lett. **B439** (1998) 112-118 (hep-ph/9807353).
15. M. Jezabek and Y. Sumino, Neutrino mixing and seesaw mechanism, Phys. Lett. **B440** (1998) 327-331 (hep-ph/9807310).
16. G. Altarelli and F. Feruglio, Phys. Lett. **B439** (1998) 112-118 (hep-ph/9807353).

17. B. Stech, Are the neutrino masses and mixings closely related to the masses and mixings of quarks?, talk at 23rd Johns Hopkins Workshop on Current Problems in Particle Theory: Neutrinos in the Next Millenium, Baltimore, MD, 10-12 June 1999 (hep-ph/9909268). M. Bando, T. Kugo, and K. Yoshioki, Neutrino mass textures with large mixing, Phys. Rev. Lett. **80** (1998) 3004-3007 (hep-ph/9710417). M. Abud, F. Buccella, D. Falcone, G. Ricciardi, and F. Tramontano, Neutrino masses and mixings in  $SO(10)$ , DSF-T-99-36 (hep-ph/9911238).
18. Q. Shafi and Z. Tavartkiladze, Proton decay, neutrino oscillations and other consequences from supersymmetric  $SU(6)$  with pseudogoldstone Higgs, BA-99-39 (hep-ph/9905202); D.P. Roy, talk at 6th Topical Seminar on Neutrino and AstroParticle Physics, San Miniato, Italy, 17-21 May 1999 (hep-ph/9908262).
19. M. Fukugita, M. Tanimoto, and T. Yanagida, Atmospheric neutrino oscillation and a phenomenological lepton mass matrix, Phys. Rev. **D57** (1998) 4429-4432 (hep-ph/9709388); M. Tanimoto, Vacuum neutrino oscillations of solar neutrinos and lepton mass matrix, Phys. Rev. **D59** (1999) 017304 (hep-ph/9807283); H. Fritzsch and Z.-z. Xing, Large leptonic flavor mixing and the mass spectrum of leptons, Phys. Lett. **B440** (1998) 313-318 (hep-ph/9808272); S.K. Kang and C.S. Kim, Bimaximal lepton flavor mixing and neutrino oscillation, Phys. Rev. **D59** (1999) 091302 (hep-ph/9811379).
20. K.S. Babu and S.M. Barr, Phys. Lett. **B381** (1996) 202 (hep-ph/9511446).
21. S.M. Barr, Phys. Rev. **D55** (1997) 1659 (hep-ph/9607419).
22. J. Sato and T. Yanagida, Large lepton mixing in a coset space family unification on  $E(7)/SU(5) \times U(1)^3$ , Phys. Lett. **B430** (1998) 127-131 (hep-ph/9710516).
23. C.H. Albright, K.S. Babu, and S.M. Barr, Phys. Rev. Lett. **81** (1998) 1167 (hep-ph/9802314); C.H. Albright and S.M. Barr, Fermion masses in  $SO(10)$  with a single adjoint Higgs field, Phys. Rev. **D58** (1998) 013002 (hep-ph/9712488).
24. N. Irges, S. Lavignac, and P. Ramond, Predictions from an anomalous  $U(1)$  model of Yukawa hierarchies, Phys. Rev. **D58** (1998) 035003 (hep-ph/9802334).
25. Y. Nomura and T. Yanagida, Bimaximal neutrino mixing in  $SO(10)$  (GUT), Phys. Rev. **D59** (1999) 017303 (hep-ph/9807325); Z. Berezhiani and A. Rossi, Grand unified textures for neutrino and quark mixings, JHEP 9903:002 (1999) (hep-ph/9811447); K. Hagiwara and N. Okamura, Quark and lepton flavor mixings in the  $SU(5)$  grand unification theory, Nucl. Phys. **B548** (1999) 60-86 (hep-ph/9811495); G. Altarelli and F. Feruglio, A simple grand unification view of neutrino mixing and fermion mass matrices, Phys. Lett. **B451** (1999) 388-396 (hep-ph/9812475); K.S. Babu, J. Pati, and F. Wilczek, Fermion masses, neutrino oscillations and proton decay in the light of SuperKamiokande, (hep-ph/9812538); R. Barbieri, L.J. Hall, G.L. Kane, and G.G. Ross, Nearly degenerate neutrinos and broken flavor symmetry, OUTP-9901-P (hep-ph/9901228); K.I. Izawa, K. Kurosawa, Y. Nomura, and T. Yanagida, Grand unification scale generation through anomalous  $U(1)$  breaking, Phys. Rev. **D60** (1999) 115016 (hep-ph/9904303); E. Ma, Permutation symmetry for neutrino and charged lepton mass matrices, Phys. Rev. **D61** (2000) 033012 (hep-ph/9909249); Q. Shafi and Z. Tavartkiladze, Bimaximal neutrino mixings and proton decay in  $SO(10)$  with anomalous flavor

- $U(1)$ , BA-99-63 (hep-ph/9910314); P. Frampton and A. Rasin, Non-abelian discrete symmetries, fermion mass textures and large neutrino mixing, IFP-777-UNC (hep-ph/9910522).
26. H. Georgi and S.L. Glashow, Phys. Lett. **B86** (1979) 297.
  27. S.M. Barr and S. Raby, Minimal  $SO(10)$  unification, Phys. Rev. Lett. **79** (1998) 4748-4751.
  28. S. Dimopoulos and F. Wilczek, report No. NSF-ITP-82-07 (1981), in *The unity of fundamental interactions* Proceedings of the 19th Course of the International School of Subnuclear Physics, Erice, Italy, 1981 ed. A. Zichichi (Plenum Press, New York, 1983); K.S. Babu and S.M. Barr, Phys. Rev. **D48** (1993) 5354 (hep-ph/9306242); K.S. Babu and S.M. Barr, Phys. Rev. **D50** (1994) 3529 (hep-ph/9402291).
  29. C.H. Albright and S.M. Barr, Phys. Lett. **B452** (1999) 287 (hep-ph/9901318).
  30. C.H. Albright and S.M. Barr, minimal Higgs model, Phys. Lett. **B461** (1999) 218 (hep-ph/9906297).
  31. S.M. Barr and Ilja Dorsner, hep-ph/0003058.